The zeta function of $\mathfrak{g}_{147A}$ counting ideals

1 Presentation

$\mathfrak{g}_{147A}$ has presentation

$$\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \mid [x_1, x_2] = x_4, [x_1, x_3] = x_5, [x_1, x_6] = x_7, [x_2, x_5] = x_7, [x_3, x_4] = x_7 \rangle.$$  

$\mathfrak{g}_{147A}$ has nilpotency class 3.

2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\zeta_{\mathfrak{g}_{147A}, p}(s) = \zeta_p(s)\zeta_p(s - 1)\zeta_p(s - 2)\zeta_p(s - 3)\zeta_p(3s - 4)\zeta_p(5s - 8)$$

$$\times \zeta_p(7s - 6)\zeta_p(6s - 8)^{-1}.$$  

$\zeta_{\mathfrak{g}_{147A}}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta_{\mathfrak{g}_{147A}, p}(s) \bigg|_{p \to p^{-1}} = -p^{21 - 16s} \zeta_{\mathfrak{g}_{147A}, p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathfrak{g}_{147A}}(s)$ is 4, with a simple pole at $s = 4$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta_{\mathfrak{g}_{147A}}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$. 

7 Notes

This ideal zeta function is identical to that of $\mathfrak{g}_{147B}$, though the Lie rings themselves are non-isomorphic.