

# The zeta function of $\mathfrak{g}_{157}$ counting ideals

## 1 Presentation

$\mathfrak{g}_{157}$  has presentation

$$\left\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \mid \begin{array}{l} [x_1, x_2] = x_3, [x_1, x_3] = x_7, \\ [x_2, x_4] = x_7, [x_5, x_6] = x_7 \end{array} \right\rangle.$$

$\mathfrak{g}_{157}$  has nilpotency class 3.

## 2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\zeta_{\mathfrak{g}_{157,p}}^{\triangleleft}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(s-4)\zeta_p(3s-5)\zeta_p(7s-6).$$

$\zeta_{\mathfrak{g}_{157}}^{\triangleleft}(s)$  is uniform.

## 3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta_{\mathfrak{g}_{157,p}}^{\triangleleft}(s) \Big|_{p \rightarrow p^{-1}} = -p^{21-15s} \zeta_{\mathfrak{g}_{157,p}}^{\triangleleft}(s).$$

## 4 Abscissa of convergence and order of pole

The abscissa of convergence of  $\zeta_{\mathfrak{g}_{157}}^{\triangleleft}(s)$  is 5, with a simple pole at  $s = 5$ .

## 5 Ghost zeta function

This zeta function is its own ghost.

## 6 Natural boundary

$\zeta_{\mathfrak{g}_{157}}^{\triangleleft}(s)$  has meromorphic continuation to the whole of  $\mathbb{C}$ .