The zeta function of $\mathfrak{g}_{257A}$ counting ideals

1 Presentation

$\mathfrak{g}_{257A}$ has presentation

$$\left\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \,\middle|\, \begin{array}{c} [x_1, x_2] = x_3, [x_1, x_3] = x_6, \\ [x_1, x_5] = x_7, [x_2, x_4] = x_6 \end{array} \right\rangle.$$

$\mathfrak{g}_{257A}$ has nilpotency class 3.

2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\zeta^{\triangleleft}_{\mathfrak{g}_{257A},p}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(3s-5)\zeta_p(5s-6)\zeta_p(5s-8) \\ \times \zeta_p(7s-9)W(p, p^{-s})$$

where $W(X, Y)$ is

$$1 + X^4Y^3 - X^9Y^8 - X^{13}Y^{10}.$$

$\zeta^{\triangleleft}_{\mathfrak{g}_{257A}}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies no functional equation.

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathfrak{g}_{257A}}(s)$ is 4, with a simple pole at $s = 4$.

5 Ghost zeta function

The ghost zeta function is the product over all primes of

$$\zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(3s-5)\zeta_p(5s-6)\zeta_p(5s-8)\zeta_p(7s-9) \\ \times W_1(p, p^{-s})W_2(p, p^{-s})$$

where

$$W_1(X, Y) = 1 + X^4Y^3,$$
$$W_2(X, Y) = 1 - X^9Y^7.$$

The ghost is friendly.

6 Natural boundary

$\zeta^{\triangleleft}_{\mathfrak{g}_{257A}}(s)$ has a natural boundary at $\Re(s) = 4/3$, and is of type III.

7 Notes

This ideal zeta function is identical to that of $\mathfrak{g}_{257C}$, though the Lie rings themselves are non-isomorphic.