The zeta function of $g_{27B}$ counting ideals

1 Presentation

$g_{27B}$ has presentation

$$\left\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \left| \begin{array}{l} [x_1, x_2] = x_6, [x_1, x_5] = x_7, \\ [x_2, x_3] = x_7, [x_3, x_4] = x_6 \end{array} \right. \right. .$$

$g_{27B}$ has nilpotency class 2.

2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\zeta_{g_{27B},p}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(s-4)\zeta_p(5s-5)\zeta_p(5s-6) \times \zeta_p(7s-10)\zeta_p(10s-10)^{-1}.$$ 

$\zeta_{g_{27B}}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta_{g_{27B},p}(s)\bigg|_{p \to p^{-1}} = -p^{21-12s}\zeta_{g_{27B},p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{g_{27B}}(s)$ is 5, with a simple pole at $s = 5$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta_{g_{27B}}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.