The zeta function of H counting all subrings

1 Presentation

H has presentation

$$\langle x, y, z \mid [x, y] = z \rangle .$$

H has nilpotency class 2.

2 The local zeta function

The local zeta function was first calculated by Grunewald, Segal & Smith. It is

$$\zeta_{H,p}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(2s-2)\zeta_p(2s-3)\zeta_p(3s-3)^{-1}.$$

$\zeta_H(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\left.\zeta_{H,p}(s)\right|_{p \to p^{-1}} = -p^{3-3s}\zeta_{H,p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_H(s)$ is 2, with a double pole at $s = 2$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta_H(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.