The zeta function of $H \times L_{\mathrm{W}}$ counting ideals

1 Presentation

$H \times L_{\mathrm{W}}$ has presentation

$$\langle t, z, u, w_1, w_2, v, x_1, x_2, y \mid [t, u] = v, [z, w_1] = x_1, [z, w_2] = x_2, [z, x_1] = y \rangle .$$

$H \times L_{\mathrm{W}}$ has nilpotency class 3.

2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$
\begin{aligned}
\zeta_{H \times L_{\mathrm{W}}, p}^{\triangleleft}(s) = {} & \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(s-4)\zeta_p(3s-5)\zeta_p(3s-6) \\
& \times \zeta_p(4s-6)\zeta_p(5s-7)\zeta_p(5s-10)\zeta_p(6s-7)\zeta_p(6s-10) \\
& \times \zeta_p(7s-8)\zeta_p(7s-12)\zeta_p(8s-12)\zeta_p(9s-14)\zeta_p(9s-17) \\
& \times \zeta_p(11s-19)\zeta_p(13s-20)\zeta_p(13s-23)W(p, p^{-s})
\end{aligned}
$$

where $W(X, Y)$ is

$$
\begin{aligned}
& 1 + X^5Y^3 - X^5Y^5 - X^6Y^5 + X^7Y^5 - X^6Y^6 + X^6Y^7 - 2X^7Y^7 - 2X^{10}Y^7 \\
& - X^{11}Y^7 + X^{12}Y^7 - X^{10}Y^8 - X^{12}Y^8 + X^{10}Y^9 + X^{11}Y^9 - 4X^{12}Y^9 \\
& - X^{15}Y^9 - X^{16}Y^9 - X^{13}Y^{10} + X^{16}Y^{10} - 2X^{17}Y^{10} + 2X^{12}Y^{11} - X^{14}Y^{11} \\
& + X^{15}Y^{11} + 3X^{16}Y^{11} - 6X^{17}Y^{11} + 2X^{13}Y^{12} - X^{14}Y^{12} + 3X^{17}Y^{12} \\
& - 2X^{19}Y^{12} - X^{22}Y^{12} + X^{14}Y^{13} - X^{16}Y^{13} + 7X^{17}Y^{13} - 4X^{19}Y^{13} \\
& + X^{20}Y^{13} + X^{21}Y^{13} - 3X^{22}Y^{13} + 4X^{18}Y^{14} + 3X^{19}Y^{14} - 2X^{20}Y^{14} \\
& + 4X^{22}Y^{14} + X^{23}Y^{14} - 3X^{24}Y^{14} - X^{17}Y^{15} + 4X^{19}Y^{15} - X^{21}Y^{15} \\
& + 7X^{22}Y^{15} + 2X^{23}Y^{15} - 3X^{24}Y^{15} - 2X^{18}Y^{16} + X^{19}Y^{16} + 3X^{20}Y^{16} \\
& - X^{22}Y^{16} + X^{23}Y^{16} + 8X^{24}Y^{16} - 2X^{26}Y^{16} + 3X^{27}Y^{16} - X^{29}Y^{16} \\
& - 2X^{22}Y^{17} - 3X^{23}Y^{17} + 9X^{24}Y^{17} + X^{25}Y^{17} - X^{26}Y^{17} + 4X^{27}Y^{17} \\
& - X^{19}Y^{18} - X^{20}Y^{18} + X^{21}Y^{18} - 3X^{23}Y^{18} - 3X^{24}Y^{18} + 5X^{25}Y^{18} \\
& + 5X^{26}Y^{18} - 5X^{27}Y^{18} + 2X^{28}Y^{18} + 10X^{29}Y^{18} - X^{30}Y^{18} - X^{31}Y^{18} \\
& + X^{32}Y^{18} - 4X^{24}Y^{19} + 2X^{26}Y^{19} - 2X^{27}Y^{19} - 4X^{28}Y^{19} + 9X^{29}Y^{19} \\
& + X^{30}Y^{19} + X^{31}Y^{19} + X^{34}Y^{19} - 9X^{25}Y^{20} + 3X^{26}Y^{20} + 4X^{27}Y^{20} \\
& - 6X^{28}Y^{20} - 8X^{29}Y^{20} + 7X^{30}Y^{20} + 8X^{31}Y^{20} - 2X^{32}Y^{20} + X^{33}Y^{20} \\
& + 5X^{34}Y^{20} - 2X^{26}Y^{21} - 2X^{27}Y^{21} + 3X^{28}Y^{21} - 9X^{29}Y^{21} - 4X^{30}Y^{21}
\end{aligned}
$$

$$+ 4X^{31}Y^{21} + X^{32}Y^{21} - 3X^{33}Y^{21} + 2X^{34}Y^{21} + 3X^{36}Y^{21} + 2X^{25}Y^{22}$$
$$- 2X^{26}Y^{22} - X^{27}Y^{22} + X^{28}Y^{22} - 2X^{29}Y^{22} - 9X^{30}Y^{22} - 7X^{31}Y^{22}$$
$$+ 7X^{32}Y^{22} - 2X^{33}Y^{22} - 8X^{34}Y^{22} + 2X^{35}Y^{22} + 11X^{36}Y^{22} - 3X^{37}Y^{22}$$
$$+ 2X^{38}Y^{22} + X^{39}Y^{22} + X^{29}Y^{23} + 2X^{30}Y^{23} - 5X^{31}Y^{23} - 7X^{32}Y^{23}$$
$$+ 6X^{33}Y^{23} - 9X^{34}Y^{23} - 7X^{35}Y^{23} - X^{36}Y^{23} + 4X^{37}Y^{23} - X^{38}Y^{23}$$
$$- X^{39}Y^{23} + 2X^{40}Y^{23} + X^{41}Y^{23} + X^{29}Y^{24} + 3X^{30}Y^{24} - 2X^{31}Y^{24}$$
$$- 8X^{32}Y^{24} + 2X^{33}Y^{24} + X^{34}Y^{24} - 9X^{35}Y^{24} - 15X^{36}Y^{24} + 9X^{37}Y^{24}$$
$$- 2X^{38}Y^{24} - 6X^{39}Y^{24} + 4X^{40}Y^{24} + 3X^{41}Y^{24} + X^{31}Y^{25} + 3X^{32}Y^{25}$$
$$- 4X^{33}Y^{25} + 4X^{34}Y^{25} + 8X^{35}Y^{25} - 16X^{36}Y^{25} - 12X^{37}Y^{25} + 6X^{38}Y^{25}$$
$$- 3X^{39}Y^{25} - 9X^{40}Y^{25} - X^{41}Y^{25} + 2X^{43}Y^{25} - X^{44}Y^{25} + X^{45}Y^{25}$$
$$+ X^{31}Y^{26} + 2X^{32}Y^{26} - 3X^{33}Y^{26} + X^{34}Y^{26} + 10X^{35}Y^{26} - 3X^{36}Y^{26}$$
$$- 11X^{37}Y^{26} - 3X^{38}Y^{26} + X^{39}Y^{26} - 5X^{40}Y^{26} - 14X^{41}Y^{26} + 5X^{42}Y^{26}$$
$$- X^{44}Y^{26} + X^{45}Y^{26} + 2X^{33}Y^{27} - X^{34}Y^{27} - X^{35}Y^{27} + 8X^{36}Y^{27}$$
$$+ 7X^{37}Y^{27} - 12X^{38}Y^{27} + X^{39}Y^{27} + 12X^{40}Y^{27} - 16X^{41}Y^{27} - 13X^{42}Y^{27}$$
$$- 2X^{43}Y^{27} + 3X^{44}Y^{27} - 6X^{45}Y^{27} - 4X^{46}Y^{27} + 3X^{47}Y^{27} - X^{35}Y^{28}$$
$$+ 3X^{36}Y^{28} + 6X^{37}Y^{28} - X^{38}Y^{28} - X^{39}Y^{28} + 5X^{40}Y^{28} + 7X^{41}Y^{28}$$
$$- 11X^{42}Y^{28} - 11X^{43}Y^{28} + 3X^{44}Y^{28} - 2X^{45}Y^{28} - 9X^{46}Y^{28} + X^{47}Y^{28}$$
$$+ 6X^{38}Y^{29} + 2X^{39}Y^{29} - 9X^{40}Y^{29} + 16X^{41}Y^{29} + 18X^{42}Y^{29} - 17X^{43}Y^{29}$$
$$- 10X^{44}Y^{29} + 15X^{45}Y^{29} - 9X^{46}Y^{29} - 14X^{47}Y^{29} - 3X^{48}Y^{29} + 2X^{49}Y^{29}$$
$$- 4X^{50}Y^{29} - X^{37}Y^{30} + X^{38}Y^{30} + X^{39}Y^{30} - X^{40}Y^{30} + 2X^{41}Y^{30}$$
$$+ 9X^{42}Y^{30} + 11X^{43}Y^{30} - 7X^{44}Y^{30} + 10X^{46}Y^{30} - 3X^{47}Y^{30} - 13X^{48}Y^{30}$$
$$+ X^{49}Y^{30} - X^{50}Y^{30} - 2X^{51}Y^{30} - X^{53}Y^{30} + 2X^{40}Y^{31} - 4X^{41}Y^{31}$$
$$- 2X^{42}Y^{31} + 19X^{43}Y^{31} + 9X^{44}Y^{31} - 22X^{45}Y^{31} + 26X^{46}Y^{31} + 19X^{47}Y^{31}$$
$$- 13X^{48}Y^{31} - 12X^{49}Y^{31} + 6X^{50}Y^{31} - 4X^{51}Y^{31} - 8X^{52}Y^{31} - 2X^{53}Y^{31}$$
$$+ X^{54}Y^{31} - X^{55}Y^{31} - X^{41}Y^{32} - 3X^{42}Y^{32} + X^{43}Y^{32} + 6X^{44}Y^{32}$$
$$+ 3X^{45}Y^{32} - X^{46}Y^{32} + 3X^{47}Y^{32} + 26X^{48}Y^{32} - 4X^{49}Y^{32} - 7X^{50}Y^{32}$$
$$+ 9X^{51}Y^{32} + 2X^{52}Y^{32} - 8X^{53}Y^{32} - 2X^{54}Y^{32} - X^{57}Y^{32} - X^{42}Y^{33}$$
$$- 4X^{43}Y^{33} + X^{44}Y^{33} + 11X^{45}Y^{33} - 11X^{46}Y^{33} - 5X^{47}Y^{33} + 35X^{48}Y^{33}$$
$$+ 10X^{49}Y^{33} - 13X^{50}Y^{33} + 8X^{51}Y^{33} + 21X^{52}Y^{33} - 8X^{53}Y^{33} - 10X^{54}Y^{33}$$
$$+ X^{55}Y^{33} + X^{56}Y^{33} - 6X^{57}Y^{33} - X^{44}Y^{34} + X^{46}Y^{34} - 5X^{47}Y^{34}$$
$$- 8X^{48}Y^{34} + 13X^{49}Y^{34} + 18X^{50}Y^{34} - 5X^{51}Y^{34} - 8X^{52}Y^{34} + 36X^{53}Y^{34}$$
$$+ 3X^{54}Y^{34} - 11X^{55}Y^{34} + 4X^{56}Y^{34} + 4X^{57}Y^{34} - 2X^{58}Y^{34} - 2X^{59}Y^{34}$$
$$- X^{44}Y^{35} - 2X^{45}Y^{35} + 2X^{46}Y^{35} - 12X^{48}Y^{35} - 2X^{49}Y^{35} + 23X^{50}Y^{35}$$
$$- 4X^{51}Y^{35} - 17X^{52}Y^{35} + 36X^{53}Y^{35} + 15X^{54}Y^{35} - 5X^{55}Y^{35} - X^{56}Y^{35}$$

$$
\begin{aligned}
&+ 14X^{57}Y^{35} - 3X^{58}Y^{35} - 6X^{59}Y^{35} + X^{60}Y^{35} - X^{61}Y^{35} + X^{47}Y^{36} \\
&- 6X^{49}Y^{36} - 10X^{50}Y^{36} + 8X^{51}Y^{36} + 8X^{52}Y^{36} - 28X^{53}Y^{36} + 9X^{54}Y^{36} \\
&+ 41X^{55}Y^{36} - 12X^{56}Y^{36} - 7X^{57}Y^{36} + 27X^{58}Y^{36} + 8X^{59}Y^{36} - 8X^{60}Y^{36} \\
&+ X^{61}Y^{36} + X^{63}Y^{36} - X^{64}Y^{36} - X^{49}Y^{37} - 8X^{50}Y^{37} + 4X^{51}Y^{37} \\
&+ 8X^{52}Y^{37} - 25X^{53}Y^{37} - 5X^{54}Y^{37} + 23X^{55}Y^{37} + 7X^{56}Y^{37} - 10X^{57}Y^{37} \\
&+ 21X^{58}Y^{37} + 10X^{59}Y^{37} + 2X^{60}Y^{37} - 2X^{61}Y^{37} + 6X^{62}Y^{37} - 2X^{64}Y^{37} \\
&+ X^{50}Y^{38} - 4X^{51}Y^{38} - 2X^{52}Y^{38} + 8X^{53}Y^{38} - 15X^{54}Y^{38} - 29X^{55}Y^{38} \\
&+ 21X^{56}Y^{38} + 16X^{57}Y^{38} - 36X^{58}Y^{38} + 4X^{59}Y^{38} + 42X^{60}Y^{38} - 3X^{61}Y^{38} \\
&- 5X^{62}Y^{38} + 11X^{63}Y^{38} + 7X^{64}Y^{38} - 2X^{65}Y^{38} - X^{66}Y^{38} + X^{67}Y^{38} \\
&- X^{51}Y^{39} - X^{52}Y^{39} + 3X^{53}Y^{39} - 3X^{54}Y^{39} - 16X^{55}Y^{39} + 8X^{57}Y^{39} \\
&- 22X^{58}Y^{39} - 4X^{59}Y^{39} + 9X^{60}Y^{39} + 15X^{61}Y^{39} - X^{62}Y^{39} + 3X^{63}Y^{39} \\
&+ 7X^{64}Y^{39} + 5X^{65}Y^{39} - X^{66}Y^{39} + X^{67}Y^{39} - X^{52}Y^{40} + 4X^{54}Y^{40} \\
&+ 2X^{55}Y^{40} - 15X^{56}Y^{40} - 11X^{57}Y^{40} + 15X^{58}Y^{40} - 6X^{59}Y^{40} - 57X^{60}Y^{40} \\
&+ 24X^{61}Y^{40} + 28X^{62}Y^{40} - 30X^{63}Y^{40} - 6X^{64}Y^{40} + 35X^{65}Y^{40} + X^{66}Y^{40} \\
&- 3X^{67}Y^{40} + 3X^{68}Y^{40} + 4X^{69}Y^{40} - X^{70}Y^{40} + X^{54}Y^{41} + X^{55}Y^{41} \\
&- 2X^{56}Y^{41} - 6X^{57}Y^{41} + 4X^{58}Y^{41} - X^{59}Y^{41} - 19X^{60}Y^{41} - 10X^{61}Y^{41} \\
&- 5X^{62}Y^{41} + X^{63}Y^{41} - 6X^{64}Y^{41} - 5X^{65}Y^{41} + 13X^{66}Y^{41} + 3X^{67}Y^{41} \\
&- 3X^{68}Y^{41} + 4X^{69}Y^{41} + 3X^{70}Y^{41} + 2X^{56}Y^{42} + X^{57}Y^{42} - 6X^{58}Y^{42} \\
&+ 3X^{59}Y^{42} + 20X^{60}Y^{42} - 35X^{61}Y^{42} - 31X^{62}Y^{42} + 33X^{63}Y^{42} - 2X^{64}Y^{42} \\
&- 71X^{65}Y^{42} + 24X^{66}Y^{42} + 22X^{67}Y^{42} - 13X^{68}Y^{42} - 4X^{69}Y^{42} + 14X^{70}Y^{42} \\
&+ 3X^{71}Y^{42} + X^{74}Y^{42} + X^{57}Y^{43} - X^{58}Y^{43} - X^{59}Y^{43} + 5X^{60}Y^{43} + X^{61}Y^{43} \\
&- 7X^{62}Y^{43} - 11X^{63}Y^{43} + 6X^{64}Y^{43} - 12X^{65}Y^{43} - 18X^{66}Y^{43} - 17X^{67}Y^{43} \\
&+ 12X^{68}Y^{43} - 3X^{69}Y^{43} - 11X^{70}Y^{43} + 5X^{71}Y^{43} + 6X^{72}Y^{43} - 2X^{73}Y^{43} \\
&+ X^{74}Y^{43} + X^{76}Y^{43} + X^{58}Y^{44} - X^{59}Y^{44} - X^{60}Y^{44} + 11X^{61}Y^{44} \\
&+ 5X^{62}Y^{44} - 21X^{63}Y^{44} - X^{64}Y^{44} + 34X^{65}Y^{44} - 40X^{66}Y^{44} - 45X^{67}Y^{44} \\
&+ 32X^{68}Y^{44} - 5X^{69}Y^{44} - 41X^{70}Y^{44} + 3X^{71}Y^{44} + 17X^{72}Y^{44} - X^{73}Y^{44} \\
&- 3X^{74}Y^{44} + 2X^{75}Y^{44} + 2X^{76}Y^{44} + X^{61}Y^{45} + 2X^{62}Y^{45} + 3X^{63}Y^{45} \\
&- 4X^{64}Y^{45} + 7X^{65}Y^{45} + 7X^{66}Y^{45} + 4X^{67}Y^{45} - 33X^{68}Y^{45} - 4X^{69}Y^{45} \\
&+ 10X^{70}Y^{45} - 17X^{71}Y^{45} - 30X^{72}Y^{45} + 17X^{73}Y^{45} - 2X^{74}Y^{45} - 9X^{75}Y^{45} \\
&+ 3X^{76}Y^{45} + 2X^{77}Y^{45} + 2X^{62}Y^{46} + 3X^{63}Y^{46} + X^{64}Y^{46} - 5X^{65}Y^{46} \\
&+ 16X^{66}Y^{46} + 19X^{67}Y^{46} - 37X^{68}Y^{46} - 6X^{69}Y^{46} + 36X^{70}Y^{46} - 28X^{71}Y^{46} \\
&- 48X^{72}Y^{46} + 16X^{73}Y^{46} + 2X^{74}Y^{46} - 19X^{75}Y^{46} + 7X^{77}Y^{46} - X^{79}Y^{46} \\
&+ X^{80}Y^{46} + 2X^{65}Y^{47} - 2X^{66}Y^{47} + 5X^{67}Y^{47} + 19X^{68}Y^{47} - 5X^{69}Y^{47} \\
&- 8X^{70}Y^{47} + 14X^{71}Y^{47} + 21X^{72}Y^{47} - 51X^{73}Y^{47} - 5X^{74}Y^{47} + 22X^{75}Y^{47}
\end{aligned}
$$

$$- 22X^{76}Y^{47} - 20X^{77}Y^{47} + 8X^{78}Y^{47} + X^{79}Y^{47} - 3X^{80}Y^{47} + X^{64}Y^{48}$$
$$- X^{66}Y^{48} + X^{67}Y^{48} + 16X^{68}Y^{48} + 3X^{69}Y^{48} - 14X^{70}Y^{48} + 18X^{71}Y^{48}$$
$$+ 35X^{72}Y^{48} - 45X^{73}Y^{48} - 6X^{74}Y^{48} + 24X^{75}Y^{48} - 16X^{76}Y^{48} - 29X^{77}Y^{48}$$
$$+ 3X^{78}Y^{48} - 3X^{80}Y^{48} - X^{81}Y^{48} - X^{82}Y^{48} + X^{83}Y^{48} - X^{67}Y^{49}$$
$$- 3X^{68}Y^{49} + 8X^{69}Y^{49} + 5X^{70}Y^{49} - 8X^{71}Y^{49} + 5X^{72}Y^{49} + 43X^{73}Y^{49}$$
$$- 31X^{75}Y^{49} + 19X^{76}Y^{49} + 28X^{77}Y^{49} - 44X^{78}Y^{49} - 9X^{79}Y^{49} + 15X^{80}Y^{49}$$
$$- 9X^{81}Y^{49} - 12X^{82}Y^{49} + 2X^{83}Y^{49} + X^{84}Y^{49} - X^{86}Y^{49} - X^{68}Y^{50}$$
$$+ 2X^{69}Y^{50} + 3X^{70}Y^{50} - 2X^{71}Y^{50} - 3X^{72}Y^{50} + 27X^{73}Y^{50} + 8X^{74}Y^{50}$$
$$- 16X^{75}Y^{50} + 12X^{76}Y^{50} + 36X^{77}Y^{50} - 27X^{78}Y^{50} - 6X^{79}Y^{50} - 2X^{81}Y^{50}$$
$$- 8X^{82}Y^{50} - 5X^{83}Y^{50} + X^{84}Y^{50} - X^{86}Y^{50} - X^{70}Y^{51} + 3X^{71}Y^{51}$$
$$- 4X^{72}Y^{51} - 9X^{73}Y^{51} + 11X^{74}Y^{51} + 26X^{75}Y^{51} - 13X^{76}Y^{51} - 4X^{77}Y^{51}$$
$$+ 52X^{78}Y^{51} + 15X^{79}Y^{51} - 46X^{80}Y^{51} + 18X^{81}Y^{51} + 25X^{82}Y^{51} - 26X^{83}Y^{51}$$
$$- 12X^{84}Y^{51} + 8X^{85}Y^{51} - 3X^{86}Y^{51} - 3X^{87}Y^{51} - 3X^{73}Y^{52} + X^{74}Y^{52}$$
$$+ 6X^{75}Y^{52} + 5X^{76}Y^{52} - 10X^{77}Y^{52} + 27X^{78}Y^{52} + 16X^{79}Y^{52} - 6X^{80}Y^{52}$$
$$+ 4X^{81}Y^{52} + 18X^{82}Y^{52} - 3X^{83}Y^{52} - 7X^{84}Y^{52} - 5X^{85}Y^{52} + X^{86}Y^{52}$$
$$+ X^{87}Y^{52} - 3X^{88}Y^{52} - X^{89}Y^{52} - 3X^{74}Y^{53} - 5X^{75}Y^{53} + 8X^{76}Y^{53}$$
$$- X^{77}Y^{53} - 18X^{78}Y^{53} + 10X^{79}Y^{53} + 53X^{80}Y^{53} - 16X^{81}Y^{53} - 16X^{82}Y^{53}$$
$$+ 53X^{83}Y^{53} + 17X^{84}Y^{53} - 41X^{85}Y^{53} + 13X^{86}Y^{53} + 11X^{87}Y^{53} - 10X^{88}Y^{53}$$
$$- 4X^{89}Y^{53} - X^{74}Y^{54} - X^{76}Y^{54} - 2X^{78}Y^{54} - 8X^{79}Y^{54} + 12X^{80}Y^{54}$$
$$+ 8X^{81}Y^{54} - 5X^{82}Y^{54} + 11X^{83}Y^{54} + 22X^{84}Y^{54} + 10X^{85}Y^{54} - 4X^{86}Y^{54}$$
$$+ 2X^{87}Y^{54} + 8X^{88}Y^{54} - X^{89}Y^{54} - 5X^{90}Y^{54} + X^{91}Y^{54} + X^{92}Y^{54}$$
$$- X^{93}Y^{54} - 2X^{76}Y^{55} - X^{77}Y^{55} + 2X^{78}Y^{55} - 9X^{79}Y^{55} - 16X^{80}Y^{55}$$
$$+ 18X^{81}Y^{55} + 6X^{82}Y^{55} - 24X^{83}Y^{55} + 5X^{84}Y^{55} + 62X^{85}Y^{55} - 14X^{86}Y^{55}$$
$$- 17X^{87}Y^{55} + 39X^{88}Y^{55} + 9X^{89}Y^{55} - 15X^{90}Y^{55} - 3X^{91}Y^{55} + 6X^{92}Y^{55}$$
$$- 2X^{93}Y^{55} - X^{94}Y^{55} - 3X^{80}Y^{56} - 5X^{81}Y^{56} + 2X^{83}Y^{56} - 18X^{84}Y^{56}$$
$$+ 9X^{85}Y^{56} + 13X^{86}Y^{56} + 5X^{87}Y^{56} - 4X^{88}Y^{56} + 14X^{89}Y^{56} + 21X^{90}Y^{56}$$
$$- 4X^{91}Y^{56} - 7X^{92}Y^{56} + 8X^{93}Y^{56} + 2X^{94}Y^{56} - 3X^{95}Y^{56} + X^{80}Y^{57}$$
$$- 9X^{81}Y^{57} + 4X^{83}Y^{57} - 10X^{84}Y^{57} - 27X^{85}Y^{57} + 16X^{86}Y^{57} + 23X^{87}Y^{57}$$
$$- 27X^{88}Y^{57} + X^{89}Y^{57} + 49X^{90}Y^{57} - X^{91}Y^{57} - 14X^{92}Y^{57} + 14X^{93}Y^{57}$$
$$+ 8X^{94}Y^{57} - 5X^{95}Y^{57} - X^{96}Y^{57} + X^{97}Y^{57} + X^{82}Y^{58} - 4X^{83}Y^{58}$$
$$+ 2X^{84}Y^{58} - 4X^{85}Y^{58} - 14X^{86}Y^{58} - 5X^{87}Y^{58} + 12X^{88}Y^{58} - 24X^{89}Y^{58}$$
$$- 6X^{90}Y^{58} + 18X^{91}Y^{58} + 22X^{92}Y^{58} - 16X^{93}Y^{58} + 3X^{94}Y^{58} + 20X^{95}Y^{58}$$
$$- 4X^{96}Y^{58} - 3X^{97}Y^{58} + 2X^{98}Y^{58} + X^{99}Y^{58} - X^{83}Y^{59} + X^{84}Y^{59}$$
$$- 11X^{86}Y^{59} - 11X^{87}Y^{59} + 8X^{88}Y^{59} - 6X^{89}Y^{59} - 33X^{90}Y^{59} + 8X^{91}Y^{59}$$

$$+ 33X^{92}Y^{59} - 22X^{93}Y^{59} - 2X^{94}Y^{59} + 25X^{95}Y^{59} + 4X^{96}Y^{59} - 3X^{97}Y^{59}$$
$$+ 2X^{98}Y^{59} + 4X^{86}Y^{60} + X^{87}Y^{60} - 15X^{88}Y^{60} + 10X^{89}Y^{60} - 8X^{90}Y^{60}$$
$$- 24X^{91}Y^{60} - 11X^{92}Y^{60} + 21X^{93}Y^{60} - 16X^{94}Y^{60} - 19X^{95}Y^{60} + 20X^{96}Y^{60}$$
$$+ 15X^{97}Y^{60} - 9X^{98}Y^{60} + 6X^{100}Y^{60} + 2X^{101}Y^{60} - X^{102}Y^{60} + X^{86}Y^{61}$$
$$- 4X^{88}Y^{61} + 2X^{90}Y^{61} - 11X^{91}Y^{61} - 21X^{92}Y^{61} + 10X^{93}Y^{61} - 5X^{94}Y^{61}$$
$$- 26X^{95}Y^{61} + X^{96}Y^{61} + 22X^{97}Y^{61} - 6X^{98}Y^{61} - 2X^{99}Y^{61} + 6X^{100}Y^{61}$$
$$+ 3X^{101}Y^{61} + X^{102}Y^{61} - X^{103}Y^{61} + 2X^{88}Y^{62} + X^{90}Y^{62} + 9X^{91}Y^{62}$$
$$- 7X^{92}Y^{62} - 19X^{93}Y^{62} + 9X^{94}Y^{62} - 31X^{96}Y^{62} - 18X^{97}Y^{62} + 19X^{98}Y^{62}$$
$$+ X^{99}Y^{62} - 22X^{100}Y^{62} + 10X^{101}Y^{62} + 11X^{102}Y^{62} - 3X^{103}Y^{62}$$
$$- 2X^{104}Y^{62} + 3X^{105}Y^{62} + 3X^{91}Y^{63} - X^{92}Y^{63} - 4X^{93}Y^{63} - X^{94}Y^{63}$$
$$- X^{95}Y^{63} - X^{96}Y^{63} - 31X^{97}Y^{63} + 4X^{98}Y^{63} + 4X^{99}Y^{63} - 14X^{100}Y^{63}$$
$$- 5X^{101}Y^{63} + 5X^{102}Y^{63} + 5X^{103}Y^{63} - X^{104}Y^{63} + X^{106}Y^{63} + X^{92}Y^{64}$$
$$+ 7X^{93}Y^{64} - 2X^{94}Y^{64} + 15X^{96}Y^{64} - 8X^{97}Y^{64} - 27X^{98}Y^{64} + 15X^{100}Y^{64}$$
$$- 28X^{101}Y^{64} - 24X^{102}Y^{64} + 19X^{103}Y^{64} + 4X^{104}Y^{64} - 12X^{105}Y^{64}$$
$$+ 4X^{106}Y^{64} + 2X^{107}Y^{64} + X^{93}Y^{65} + X^{95}Y^{65} + 7X^{97}Y^{65} + X^{98}Y^{65}$$
$$- 13X^{99}Y^{65} + 4X^{100}Y^{65} + X^{101}Y^{65} - 22X^{102}Y^{65} - 6X^{103}Y^{65} + 3X^{104}Y^{65}$$
$$- 5X^{106}Y^{65} - 2X^{107}Y^{65} + 3X^{108}Y^{65} + X^{109}Y^{65} + 2X^{95}Y^{66} - X^{96}Y^{66}$$
$$+ 6X^{97}Y^{66} + 12X^{98}Y^{66} - 6X^{99}Y^{66} - 7X^{100}Y^{66} + 22X^{101}Y^{66} - 7X^{102}Y^{66}$$
$$- 27X^{103}Y^{66} - 6X^{104}Y^{66} + 13X^{105}Y^{66} - 13X^{106}Y^{66} - 13X^{107}Y^{66}$$
$$+ 6X^{108}Y^{66} + 3X^{109}Y^{66} - X^{110}Y^{66} - 2X^{111}Y^{66} + X^{112}Y^{66} + X^{97}Y^{67}$$
$$+ 4X^{99}Y^{67} + 2X^{100}Y^{67} - 2X^{101}Y^{67} + 12X^{102}Y^{67} + 6X^{103}Y^{67}$$
$$- 20X^{104}Y^{67} + X^{105}Y^{67} + 7X^{106}Y^{67} - 10X^{107}Y^{67} - 9X^{108}Y^{67}$$
$$- 3X^{109}Y^{67} + 3X^{110}Y^{67} - 2X^{112}Y^{67} + X^{113}Y^{67} + 2X^{99}Y^{68} + 5X^{100}Y^{68}$$
$$- 5X^{101}Y^{68} + 14X^{102}Y^{68} + 11X^{103}Y^{68} - 4X^{104}Y^{68} - 9X^{105}Y^{68}$$
$$+ 10X^{106}Y^{68} + 8X^{107}Y^{68} - 22X^{108}Y^{68} - 10X^{109}Y^{68} + 10X^{110}Y^{68}$$
$$- 4X^{111}Y^{68} - 5X^{112}Y^{68} + 2X^{113}Y^{68} + X^{101}Y^{69} - 4X^{103}Y^{69} + 15X^{104}Y^{69}$$
$$+ 3X^{105}Y^{69} - 6X^{106}Y^{69} + 13X^{107}Y^{69} + 11X^{108}Y^{69} - 14X^{109}Y^{69}$$
$$- 8X^{110}Y^{69} + 6X^{111}Y^{69} + X^{112}Y^{69} - 8X^{113}Y^{69} - X^{114}Y^{69} + X^{115}Y^{69}$$
$$- X^{102}Y^{70} + 8X^{104}Y^{70} + 4X^{105}Y^{70} - 4X^{106}Y^{70} + 6X^{107}Y^{70} + 17X^{108}Y^{70}$$
$$+ 2X^{109}Y^{70} - 12X^{110}Y^{70} + 3X^{111}Y^{70} + 7X^{112}Y^{70} - 12X^{113}Y^{70}$$
$$- 3X^{114}Y^{70} + X^{115}Y^{70} - X^{117}Y^{70} - 2X^{104}Y^{71} - X^{105}Y^{71} + 4X^{106}Y^{71}$$
$$- X^{107}Y^{71} - 7X^{108}Y^{71} + 27X^{109}Y^{71} + 4X^{110}Y^{71} - 13X^{111}Y^{71}$$
$$+ 8X^{112}Y^{71} + 13X^{113}Y^{71} - 4X^{114}Y^{71} - 8X^{115}Y^{71} + X^{116}Y^{71} + 2X^{117}Y^{71}$$
$$- 2X^{118}Y^{71} - X^{119}Y^{71} - X^{105}Y^{72} + 2X^{106}Y^{72} - 2X^{107}Y^{72} - X^{108}Y^{72}$$

$$+ 11X^{109}Y^{72} + 4X^{110}Y^{72} + 2X^{111}Y^{72} - X^{112}Y^{72} + 12X^{113}Y^{72}$$
$$+ 8X^{114}Y^{72} - 9X^{115}Y^{72} - 4X^{116}Y^{72} + 4X^{117}Y^{72} - 2X^{118}Y^{72} - X^{119}Y^{72}$$
$$- X^{106}Y^{73} - 8X^{109}Y^{73} + 14X^{111}Y^{73} - 6X^{112}Y^{73} - 6X^{113}Y^{73}$$
$$+ 21X^{114}Y^{73} + 9X^{115}Y^{73} - 7X^{116}Y^{73} - X^{117}Y^{73} + 6X^{118}Y^{73} + 4X^{119}Y^{73}$$
$$- 6X^{120}Y^{73} - 2X^{109}Y^{74} + 2X^{111}Y^{74} + X^{112}Y^{74} - 2X^{113}Y^{74} + 4X^{114}Y^{74}$$
$$+ 5X^{115}Y^{74} + 9X^{116}Y^{74} - 5X^{117}Y^{74} + 6X^{118}Y^{74} + 3X^{119}Y^{74} - 2X^{120}Y^{74}$$
$$- X^{121}Y^{74} - X^{110}Y^{75} - 3X^{111}Y^{75} + X^{113}Y^{75} - 12X^{114}Y^{75} - X^{115}Y^{75}$$
$$+ 18X^{116}Y^{75} - 2X^{117}Y^{75} - 9X^{118}Y^{75} + 10X^{119}Y^{75} + 10X^{120}Y^{75}$$
$$- 2X^{121}Y^{75} - X^{122}Y^{75} + X^{123}Y^{75} + X^{124}Y^{75} - X^{125}Y^{75} - 3X^{114}Y^{76}$$
$$+ X^{115}Y^{76} - 3X^{116}Y^{76} + 3X^{117}Y^{76} + 2X^{118}Y^{76} - 4X^{119}Y^{76} + 5X^{120}Y^{76}$$
$$+ 9X^{121}Y^{76} - 3X^{122}Y^{76} + X^{123}Y^{76} + X^{124}Y^{76} - X^{115}Y^{77} - 10X^{116}Y^{77}$$
$$+ 3X^{117}Y^{77} + 5X^{118}Y^{77} - 12X^{119}Y^{77} - 3X^{120}Y^{77} + 15X^{121}Y^{77}$$
$$- 3X^{123}Y^{77} + 6X^{125}Y^{77} - X^{116}Y^{78} - 2X^{118}Y^{78} - 2X^{120}Y^{78} - 10X^{121}Y^{78}$$
$$+ 7X^{122}Y^{78} + 3X^{123}Y^{78} - 3X^{124}Y^{78} + X^{125}Y^{78} + 4X^{126}Y^{78} + X^{116}Y^{79}$$
$$- 2X^{117}Y^{79} - X^{118}Y^{79} + X^{119}Y^{79} - X^{120}Y^{79} - 14X^{121}Y^{79} + 4X^{122}Y^{79}$$
$$+ 6X^{123}Y^{79} - 5X^{124}Y^{79} - 4X^{125}Y^{79} + 3X^{126}Y^{79} + 3X^{127}Y^{79} - X^{129}Y^{79}$$
$$+ 2X^{130}Y^{79} + X^{121}Y^{80} - 2X^{122}Y^{80} - 6X^{123}Y^{80} + 2X^{124}Y^{80} - X^{125}Y^{80}$$
$$- 8X^{126}Y^{80} + 3X^{127}Y^{80} + 4X^{128}Y^{80} - X^{129}Y^{80} - X^{130}Y^{80} + X^{131}Y^{80}$$
$$+ X^{121}Y^{81} - X^{122}Y^{81} - 5X^{123}Y^{81} + 3X^{124}Y^{81} - X^{125}Y^{81} - 9X^{126}Y^{81}$$
$$+ 2X^{128}Y^{81} - 2X^{130}Y^{81} + X^{132}Y^{81} + 2X^{123}Y^{82} - 2X^{124}Y^{82} + X^{125}Y^{82}$$
$$+ 3X^{126}Y^{82} - 3X^{127}Y^{82} - 9X^{128}Y^{82} + 3X^{129}Y^{82} - 5X^{131}Y^{82} + 2X^{132}Y^{82}$$
$$+ 3X^{126}Y^{83} - 3X^{127}Y^{83} - 5X^{128}Y^{83} + X^{129}Y^{83} - 2X^{131}Y^{83} - 2X^{132}Y^{83}$$
$$+ 2X^{127}Y^{84} + 3X^{128}Y^{84} - 3X^{129}Y^{84} + 3X^{131}Y^{84} - 3X^{132}Y^{84} - 5X^{133}Y^{84}$$
$$+ X^{134}Y^{84} - X^{136}Y^{84} + X^{128}Y^{85} - X^{129}Y^{85} + X^{131}Y^{85} - 3X^{133}Y^{85}$$
$$- X^{137}Y^{85} + X^{129}Y^{86} + X^{131}Y^{86} + 5X^{133}Y^{86} - 4X^{134}Y^{86} - X^{135}Y^{86}$$
$$+ 2X^{136}Y^{86} - X^{137}Y^{86} - 3X^{138}Y^{86} + X^{139}Y^{86} + 2X^{133}Y^{87} - X^{134}Y^{87}$$
$$+ X^{138}Y^{87} - X^{139}Y^{87} + 2X^{134}Y^{88} + X^{135}Y^{88} + X^{137}Y^{88} + 3X^{138}Y^{88}$$
$$- 2X^{139}Y^{88} + X^{138}Y^{89} + X^{139}Y^{89} + X^{139}Y^{90} + X^{140}Y^{90} + 2X^{143}Y^{90}$$
$$- X^{144}Y^{90} + X^{140}Y^{91} + X^{144}Y^{91} - X^{143}Y^{92} + 2X^{144}Y^{92} - X^{144}Y^{93}$$
$$+ X^{145}Y^{93} - X^{145}Y^{95} - X^{150}Y^{97}.$$

$\zeta^{\lhd}_{H \times L_{\mathrm{W}}}(s)$ is uniform.

3   Functional equation

The local zeta function satisfies no functional equation.

4   Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{H \times L_{\mathrm{W}}}^{\lhd}(s)$ is 5, with a simple pole at $s = 5$.

5   Ghost zeta function

The ghost zeta function is the product over all primes of

$$\zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(s-4)\zeta_p(3s-5)\zeta_p(3s-6)\zeta_p(4s-6)$$
$$\times\, \zeta_p(5s-7)\zeta_p(5s-10)\zeta_p(6s-7)\zeta_p(6s-10)\zeta_p(7s-8)\zeta_p(7s-12)\zeta_p(8s-12)$$
$$\times\, \zeta_p(9s-14)\zeta_p(9s-17)\zeta_p(11s-19)\zeta_p(13s-20)\zeta_p(13s-23)W_1(p,p^{-s})$$
$$\times\, W_2(p,p^{-s})W_3(p,p^{-s})W_4(p,p^{-s})W_5(p,p^{-s})W_6(p,p^{-s})W_7(p,p^{-s})$$
$$\times\, W_8(p,p^{-s})W_9(p,p^{-s})W_{10}(p,p^{-s})$$

where

$$W_1(X,Y) = 1 - X^{22}Y^{12},$$
$$W_2(X,Y) = -1 + X^{23}Y^{13},$$
$$W_3(X,Y) = 1 - X^{19}Y^{11},$$
$$W_4(X,Y) = -1 + X^{12}Y^{7},$$
$$W_5(X,Y) = 1 - X^{10}Y^{6},$$
$$W_6(X,Y) = -1 + X^{26}Y^{17},$$
$$W_7(X,Y) = 1 - X^{13}Y^{9},$$
$$W_8(X,Y) = -1 + X^{14}Y^{11},$$
$$W_9(X,Y) = 1 - X^{5}Y^{4},$$
$$W_{10}(X,Y) = -1 - X^{6}Y^{7}.$$

The ghost is friendly.

6   Natural boundary

$\zeta_{H \times L_{\mathrm{W}}}^{\lhd}(s)$ has a natural boundary at $\Re(s) = 11/6$, and is of type III.

7   Notes

Since L_{W} is a direct factor, it is unsurprising that there is no functional equation here.