The zeta function of $H \times Q_5$ counting ideals

1 Presentation

$H \times Q_5$ has presentation

$$\langle t, u, x_1, x_2, v, x_3, x_4, x_5 \mid [t,u] = v, [x_1,x_2] = x_3, [x_1,x_3] = x_5, [x_2,x_4] = x_5 \rangle .$$

$H \times Q_5$ has nilpotency class 3.

2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\begin{aligned}\zeta^{\triangleleft}_{H\times Q_5,p}(s) = {} & \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(s-4)\zeta_p(3s-5)^2\zeta_p(5s-6)^2 \\ & \times \zeta_p(7s-7)\zeta_p(5s-5)^{-1}\zeta_p(7s-6)^{-1}.\end{aligned}$$

$\zeta^{\triangleleft}_{H\times Q_5}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta^{\triangleleft}_{H\times Q_5,p}(s)\big|_{p\rightarrow p^{-1}} = p^{28-16s}\zeta^{\triangleleft}_{H\times Q_5,p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{H\times Q_5}(s)$ is 5, with a simple pole at $s = 5$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta^{\triangleleft}_{H\times Q_5}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.