The zeta function of M_3 counting ideals

1 Presentation

M_3 has presentation

$$\langle z, x_1, x_2, x_3 \mid [z, x_1] = x_2, [z, x_2] = x_3 \rangle .$$

M_3 has nilpotency class 3.

2 The local zeta function

The local zeta function was first calculated by Gareth Taylor. It is

$$\zeta^{\triangleleft}_{M_3,p}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(3s-2)\zeta_p(4s-2)\zeta_p(5s-3)\zeta_p(5s-2)^{-1}.$$

$\zeta^{\triangleleft}_{M_3}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta^{\triangleleft}_{M_3,p}(s)\big|_{p\to p^{-1}} = p^{6-9s}\zeta^{\triangleleft}_{M_3,p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{M_3}(s)$ is 2, with a simple pole at $s = 2$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta^{\triangleleft}_{M_3}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.