The zeta function of $M_3$ counting ideals

1 Presentation

$M_3$ has presentation

$$\langle z, x_1, x_2, x_3 \mid [z, x_1] = x_2, [z, x_2] = x_3 \rangle.$$ 

$M_3$ has nilpotency class 3.

2 The local zeta function

The local zeta function was first calculated by Gareth Taylor. It is

$$\zeta_{M_3, p}(s) = \zeta_p(s)\zeta_p(s - 1)\zeta_p(3s - 2)\zeta_p(4s - 2)\zeta_p(5s - 3)\zeta_p(5s - 2)^{-1}.$$ 

$\zeta_{M_3}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta_{M_3, p}^\vee(s) \bigg|_{p \rightarrow p^{-1}} = p^{6 - 9s} \zeta_{M_3, p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{M_3}^\vee(s)$ is 2, with a simple pole at $s = 2$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta_{M_5}^\vee(s)$ has meromorphic continuation to the whole of $\mathbb{C}$. 