The zeta function of Q_5 counting ideals

1 Presentation

Q_5 has presentation

$$\langle x_1, x_2, x_3, x_4, x_5 \mid [x_1, x_2] = x_3, [x_1, x_3] = x_5, [x_2, x_4] = x_5 \rangle .$$

Q_5 has nilpotency class 3.

2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\zeta^{\triangleleft}_{Q_5,p}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(3s-3)\zeta_p(5s-4).$$

$\zeta^{\triangleleft}_{Q_5}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta^{\triangleleft}_{Q_5,p}(s)\Big|_{p\to p^{-1}} = -p^{10-11s}\zeta^{\triangleleft}_{Q_5,p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{Q_5}(s)$ is 3, with a simple pole at $s = 3$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta^{\triangleleft}_{Q_5}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.

7 Notes

This Lie ring comes from the only Lie algebra of dimension 5 not previously encountered by Luke Woodward. The subscript 5 comes from the dimension, but there isn't any significance in the letter Q.