The zeta function of $Q_5 \times \mathbb{Z}$ counting ideals

1 Presentation

$Q_5 \times \mathbb{Z}$ has presentation

$$\langle x_1, x_2, x_3, a, x_4, x_5 \mid [x_1, x_2] = x_4, [x_1, x_4] = x_5, [x_2, x_3] = x_5 \rangle .$$

$Q_5 \times \mathbb{Z}$ has nilpotency class 3.

2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\zeta^{\triangleleft}_{Q_5 \times \mathbb{Z},p}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(3s-4)\zeta_p(5s-5).$$

$\zeta^{\triangleleft}_{Q_5 \times \mathbb{Z}}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\left.\zeta^{\triangleleft}_{Q_5 \times \mathbb{Z},p}(s)\right|_{p \to p^{-1}} = p^{15-12s}\zeta^{\triangleleft}_{Q_5 \times \mathbb{Z},p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{Q_5 \times \mathbb{Z}}(s)$ is 4, with a simple pole at $s = 4$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta^{\triangleleft}_{Q_5 \times \mathbb{Z}}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.

7 Notes

This Lie ring comes from the only Lie algebra of dimension 5 not previously encountered by Luke Woodward. The subscript 5 comes from the dimension, but there isn't any significance in the letter Q.