The zeta function of c2mm counting normal subgroups

1 Presentation

c2mm has presentation

$$\left\langle x, y, m, r \mid [x, y], m^2, r^2, y^m = y^{-1}, x^m = xy, y^r = y^{-1}, x^r = x^{-1}, r^m = r^{-1} \right\rangle .$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\begin{aligned}\zeta^{\triangleleft}_{\mathbf{c2mm}}(s) &= 1 + 5 \cdot 2^{-s} + 2 \cdot 4^{-s} + 2 \cdot 8^{-s} + (2 \cdot 2^{-s} + 2 \cdot 4^{-s})\zeta(s) \\ &\quad + (4^{-s} - 8^{-s} + 2 \cdot 16^{-s})\zeta(s)^2.\end{aligned}$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathbf{c2mm}}(s)$ is 1, with a double pole at $s = 1$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.