The zeta function of cm counting normal subgroups

1 Presentation

cm has presentation

$$\left\langle x, y, t \mid [x, y], t^2, y^t = y^{-1}, x^t = xy \right\rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta^{\triangleleft}_{\mathbf{cm}}(s) = (1 + 2^{-s})\zeta(s) + (2^{-s} - 4^{-s} + 2 \cdot 8^{-s})\zeta(s)^2.$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathbf{cm}}(s)$ is 1, with a double pole at $s = 1$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.