

The zeta function of \mathfrak{p}_1 counting all subgroups

1 Presentation

\mathfrak{p}_1 has presentation

$$\langle x, y \mid [x, y] \rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathfrak{p}_1}(s) = \zeta(s)\zeta(s-1).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathfrak{p}_1}(s)$ is 2, with a simple pole at $s = 2$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to \mathbb{C} .