

# The zeta function of $\mathfrak{p}_1$ counting normal subgroups

## 1 Presentation

$\mathfrak{p}_1$  has presentation

$$\langle x, y \mid [x, y] \rangle.$$

## 2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathfrak{p}_1}^{\triangleleft}(s) = \zeta(s)\zeta(s-1).$$

## 3 Abscissa of convergence and order of pole

The abscissa of convergence of  $\zeta_{\mathfrak{p}_1}^{\triangleleft}(s)$  is 2, with a simple pole at  $s = 2$ . Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to  $\mathbb{C}$ .