The zeta function of p2 counting all subgroups

1 Presentation

p2 has presentation

$$\left\langle x, y, r \mid [x, y], r^2, x^r = x^{-1}, y^r = y^{-1} \right\rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathbf{p2}}(s) = \zeta(s-1)\zeta(s-2) + 2^{-s}\zeta(s)\zeta(s-1).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathbf{p2}}(s)$ is 3, with a simple pole at $s = 3$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.