## The zeta function of p2 counting normal subgroups

## 1 Presentation

p2 has presentation

$$
\left\langle x, y, r \mid[x, y], r^{2}, x^{r}=x^{-1}, y^{r}=y^{-1}\right\rangle
$$

## 2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$
\zeta_{\mathbf{p} \mathbf{2}}^{\triangleleft}(s)=1+6 \cdot 2^{-s}+4 \cdot 4^{-s}+2^{-s} \zeta(s) \zeta(s-1)
$$

## 3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathbf{p} 2}^{\triangleleft}(s)$ is 2 , with a simple pole at $s=2$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.

