The zeta function of p2gg counting normal subgroups

1 Presentation

p2gg has presentation

$$\langle x, y, u, v \mid [x, y], u^2 = x, v^2 = y, x^v = x^{-1}, y^u = y^{-1}, (uv)^2 \rangle .$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta^{\triangleleft}_{\mathbf{p2gg}}(s) = 1 + (2 \cdot 2^{-s} - 2 \cdot 4^{-s})\zeta(s) + 2^{-s} + 2 \cdot 4^{-s} + (4^{-s} + 8^{-s})\zeta(s)^2.$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathbf{p2gg}}(s)$ is 1, with a double pole at $s = 1$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.