The zeta function of p2mm counting normal subgroups

1 Presentation

p2mm has presentation

$$\left\langle x, y, p, q \mid [x,y], [p,q], p^2, q^2, x^p = x, x^q = x^{-1}, y^p = y^{-1}, y^q = y \right\rangle .$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\begin{aligned}\zeta^{\triangleleft}_{\mathbf{p2mm}}(s) &= 1 + 13 \cdot 2^{-s} + 20 \cdot 4^{-s} + 4 \cdot 8^{-s} + (2 \cdot 2^{-s} + 10 \cdot 4^{-s} + 4 \cdot 8^{-s})\zeta(s) \\ &\quad + (4^{-s} + 8^{-s})\zeta(s)^2.\end{aligned}$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathbf{p2mm}}(s)$ is 1, with a double pole at $s = 1$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.