The zeta function of p31m counting normal subgroups

1 Presentation

p31m has presentation

$$\left\langle x, y, r, t \mid [x,y], r^2, t^2, (tr)^3, x^r = x, y^t = y, x^t = x^{-1}y, y^r = xy^{-1} \right\rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathbf{p31m}}^{\triangleleft}(s) = 1 + 2^{-s} + 3^{-s} + 6^{-s} + (6^{-s} + 18^{-s})\zeta(2s).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathbf{p31m}}^{\triangleleft}(s)$ is $1/2$, with a simple pole at $s = 1/2$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.