The zeta function of p3m1 counting normal subgroups

1 Presentation

p3m1 has presentation

$$\left\langle x, y, r, m \;\middle|\; \begin{array}{c} [x,y], r^3, m^2, r^m = r^{-1}, x^r = x^{-1}y, \\ y^r = x^{-1}, x^m = x^{-1}, y^m = x^{-1}y \end{array} \right\rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta^{\triangleleft}_{\mathbf{p3m1}}(s) = 1 + 2^{-s} + 3 \cdot 6^{-s} + (6^{-s} + 18^{-s})\zeta(2s).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathbf{p3m1}}(s)$ is $1/2$, with a simple pole at $s = 1/2$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.