The zeta function of p4gm counting normal subgroups

1 Presentation

p4gm has presentation

$$\left\langle x, y, r, t \mid [x,y], r^4, t^2, y^r = x^{-1}, x^r = y, x^t = y, r^t = r^{-1}x^{-1} \right\rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta^{\triangleleft}_{\mathbf{p4gm}}(s) = 1 + 3 \cdot 2^{-s} + 3 \cdot 4^{-s} + 2 \cdot 8^{-s} + (8^{-s} + 16^{-s})\zeta(2s).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathbf{p4gm}}(s)$ is $1/2$, with a simple pole at $s = 1/2$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.