The zeta function of pg counting all subgroups

1 Presentation

pg has presentation

$$\left\langle x, y, t \mid [x, y], t^2 = x, y^t = y^{-1} \right\rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathbf{pg}}(s) = \zeta(s)\zeta(s-1).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathbf{pg}}(s)$ is 2, with a simple pole at $s = 2$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.