The zeta function of pm counting normal subgroups

1 Presentation

pm has presentation

$$\left\langle x, y, m \mid [x, y], m^2, x^m = x, y^m = y^{-1} \right\rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta^{\triangleleft}_{\mathbf{pm}}(s) = (1 + 5 \cdot 2^{-s} + 2 \cdot 4^{-s})\zeta(s) + (2^{-s} + 4^{-s})\zeta(s)^2.$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathbf{pm}}(s)$ is 1, with a double pole at $s = 1$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.